subdivision, and hence Eqs (14b) and (14c) do not apply The edge conditions supply the only applicable equations, viz, Eqs (14a) and (14d) Their solution may be written as

$$\begin{aligned} \mathbf{d}_1 &= (B_0\alpha^{-1}\beta - D_0)^{-1}(B_0\alpha^{-1}\mathbf{\Psi}^* - \mathbf{L}_0) \\ \mathbf{c}_1 &= \alpha^{-1}(\mathbf{\Psi}^* - \beta\mathbf{d}_1) \end{aligned} \quad (18)$$

After calculation of the forces and displacements $\mathbf{y}_i$, $\mathbf{z}_i$ by means of Eqs (11), shell stresses may be obtained by application of Eqs (5) and (6) (for the strains) and Hooke's law Transverse shear stresses, which may be important in bending regions,‡‡ may be obtained by application of, in addition, Eqs (4) and derivatives of Eqs (5) and (6) (for the strain derivatives) and of Hooke's law and integration of the three-dimensional equilibrium equations (with suitable simplifications)

Example

Figures 2 and 3 show meridional bending and direct stress distributions in the vicinity of the small edge of a zone of a spherical shell for which the colatitude ψ ranges from 0 4668 to $\pi/2$ rad, $R/t = 50$, and $\nu = 0$ 3 (isotropic) under the action of a tilting moment M and a lateral thrust H applied through a rigid ring at the edge $\psi = \psi = 0$ 4668 These numerical results are compared to the analytical solutions for the same problems obtained by Steele,[5] who uses the same definitions of shell strains as in the present paper§§ but assumes Love's first approximation for the elasticity relations Excellent agreement is seen at the edge, where the maximum stress conditions occurs The relatively small discrepancy in the interior apparently results from simplifications (based on order-of-magnitude considerations) made in Ref 5 in order to arrive at analytical results

In order to determine the effect of meridional and circumferential stiffening, these problems were repeated for an orthotropic material for which $\nu_1 = 0$ 3, $E_1/E_2 = 2$, and $E_1/E_{12} = 5$ 2, oriented so that the maximum stiffness direction is firstly meridional and secondly circumferential The results show that this degree of orthotropy is not great enough to alter appreciably the direct stresses (i e , the membrane stresses are still, to a good approximation, uncoupled from the bending stresses) The percentage changes in the maximum bending stresses from those of the isotropic cases are shown in Table 1

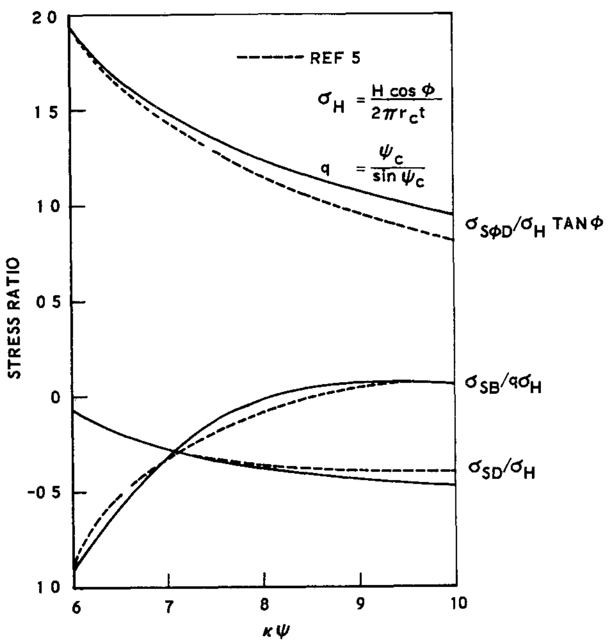

Fig 3 Stresses due to lateral thrust H

Table 1 Percentage changes in the maximum bending stresses

E/E_ϕ	M, %	H, %
2	+21	+79
½	−9 5	+58

It is noted that the six problems compared in Table 1 took a total of 6 min of Philco 2000 computer time and that two interior points of subdivision were sufficient

‡‡ The transverse normal stress need not be computed, since it is negligible even in bending regions

§§ The only disagreement occurs in the last term of his expression for κ_{12} [Eq (4f) of Ref 5], which is apparently a misprint since it is dimensionally incorrect

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Potential Flow Past a Parabolic Leading Edge

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Nomenclature

a,b	=	constants
h,s	=	coordinates of singular point
x,y,ξ,η,ξ',η'	=	rectangular coordinates
z	=	$x + iy$
ζ	=	$\xi + i\eta$
ζ'	=	$\xi' + i\eta'$
Q	=	complex potential
r	=	leading-edge radius
c	=	chord
C_p	=	pressure coefficient, $1 - [U/U_\infty]^2$
U,U_∞	=	local and remote flow speeds, respectively

Subscripts

stag	=	stagnation point
$C_{p_{\min}}$	=	minimum-pressure point

STUDY of an exact solution for the potential flow past a parabolic cylinder has led to insight into the relation between the locations of maximum and minimum pressure on an airfoil The following rule may be used to locate either of these two points when the other is known: a straight line from the stagnation point to the minimum-pressure point will always pass through a fixed third point that lies midway between the airfoil leading edge and the leading-edge center of curvature (see Fig 1)

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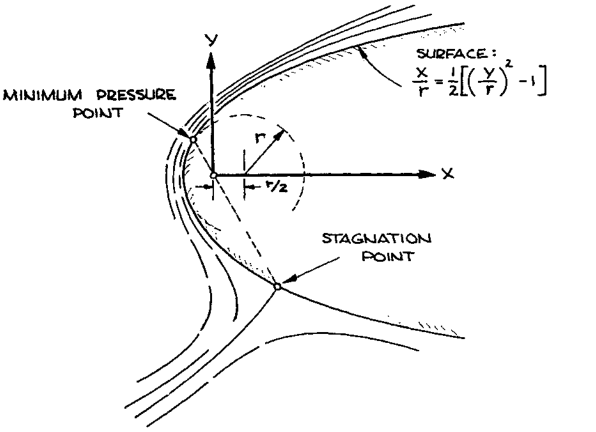

Fig 1 Relation between the locations of the stagnation point and minimum-pressure point in potential flow past a leading edge

The preceding statement is exact in the potential flow past a parabolic cylinder The rule has been found to hold to a good approximation when applied to airfoils The derivation of the exact solution is given below

Stagnation flow impinging on an infinite plane surface is represented by the potential function

$$Q = (a/2)\zeta^2 \tag{1}$$

Our procedure will be to map this flow into a flow past a leading-edge-like contour We want to locate the center of distortion beneath the surface, and so we first introduce the coordinate translation

$$\xi = \xi' - s \qquad \eta = \eta' - h \tag{2}$$

The given flow and the two coordinate systems are shown in Fig 2

To accomplish the desired mapping we will use the transformation

$$z = b(\zeta')^2 \tag{3}$$

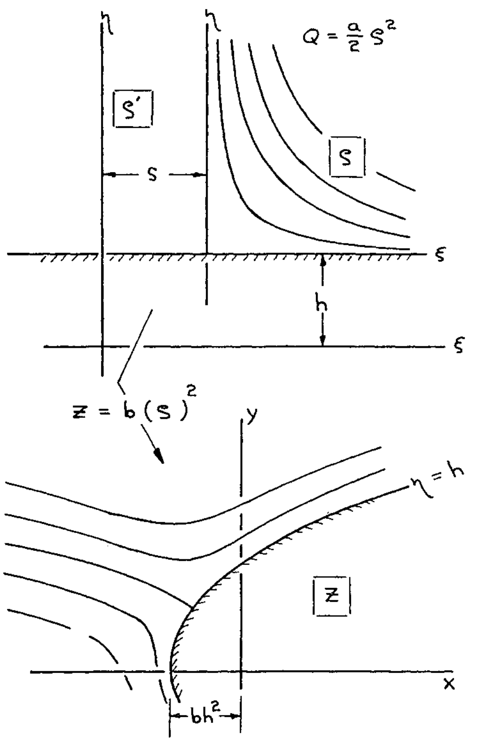

Fig 2 Conformal mapping of plane stagnation flow into the flow past a parabolic cylinder

Under this transformation the original plane wall becomes the parabolic surface

$$x/r = \tfrac{1}{2}[(y/r)^2 - 1] \tag{4}$$

After noting that $U_\infty = a/2b$, that $y_{st\ g} = 2bhs$, and that the leading-edge radius of curvature is $r = y_{\mathrm{stag}}h/s$, the pressure distribution on the parabolic cylinder becomes

$$C_p = 1 - \frac{[y/y_{\mathrm{stag}} - 1]^2}{(y/y_{\mathrm{stag}})^2 + (r/y_{\ \mathrm{tag}})^2} \tag{5}$$

From these results we obtain the following properties of the flow:

1) Location of minimum pressure:

$$y_{C_{p\min}} = -y_{\ t\ g}(r/y_{\ t\ g})^2 \tag{6}$$

2) Minimum pressure:

$$C_{p_{\min}} = -(y_{\mathrm{stag}}/r)^2 \tag{7}$$

3) Pressure at the leading edge:

$$C_{p_{LE}} = 1 - (y_{\mathrm{stag}}/r)^2 \tag{8}$$

From the first property and Eq (4) it may be shown that there exists a simple relation between the location of minimum pressure and the location of the stagnation point: a line drawn from the stagnation point to the point of minimum pressure will always pass through the focus of the parabola The relation is further illustrated in Fig 3

On an airfoil, when neither the stagnation point nor the minimum-pressure point is unusually far removed from the leading edge, the reflection rule gives good results (The rule is independent of camber) Under these same conditions, a pressure rule obtained from Eqs (7) and (8) is also found to hold:

$$C_{p_{LE}} = 1 + C_{p_{\min}} \tag{9}$$

Other useful indications obtainable from the exact solution are:

1) The familiar leading-edge pressure peak that occurs

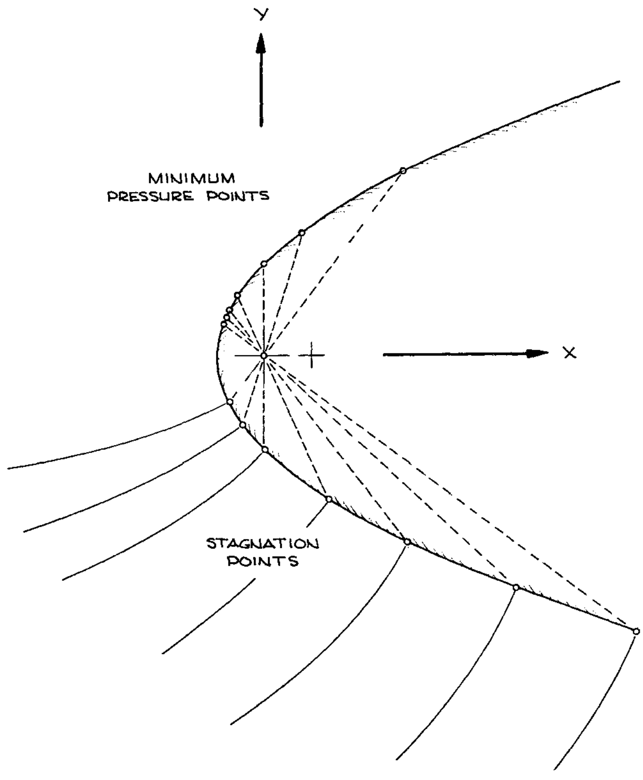

Fig 3 Exact relation between stagnation-point and minimum-pressure-point locations in potential flow past a parabolic cylinder

on an airfoil at high incidence first appears when the stagnation point moves to ordinates exceeding about one-half the leading-edge radius

2) The lift on a truncated parabolic cylinder varies linearly with the ordinate of the stagnation point Extending to airfoils, an approximate relation between stagnation-point movement and change of lift, at any constant camber, is

$$(y_{t\ g})_2/r = (y_{t\ g})_1/r + 0\ 02(c/r)(C_{L_1} - C_{L_2}) \qquad (10)$$

3) For an airfoil at its ideal angle of attack, the velocity gradient at the stagnation point is given by

$$dU/ds = U_\infty/r \qquad (11)$$

wheie s is distance along the wall In the limit, as $r \to \infty$, we know that $U_\infty \to \infty$ and $dU/ds \to a$ All possible ideal-incidence stagnation-point velocity gradients may then be summarized as $0 < r \to \infty$, $\infty > dU/ds \geq a$

These results provide a first step in the design of an airfoil from a known pressure distribution If $U = U(s)$ is known, then Eq (11) gives the airfoil leading-edge radius directly

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Ablation of a Cylindrical Cavity in an Infinite Medium

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Nomenclature

a	=	initial radius of cylindrical cavity, in
c	=	specific heat Btu/(lb-°R)
k	=	thermal conductivity, (Btu-in)/(ft²-sec-°R)
L	=	heat of sublimation, Btu/lb
Q	=	heat flux, Btu/(ft² sec)
r	=	radial coordinate in
s	=	radius of ablating heated surface, in
t	=	time, sec
T	=	temperature, °R
T_i	=	initial temperatuie, °R
T_M	=	melt or sublimation temperature, R
α	=	$(Qa)/[k(T_M - T_i)]$
β	=	$\rho L\varkappa/[k(T_M - T_i)]$
δ	=	depth of thermal layer, in
$\varkappa$	=	diffusivity, in ²/sec
ρ	=	density of solid, lb/ft³
θ	=	temperature, nondimensional, $(T - T_i)/(T_M - T_i)$
τ	=	nondimensional time $\varkappa t/a^2$
ξ,ζ	=	nondimensional space coordinates, s/a, r/a
$(\dot{\ })$	=	indicates differentiation with respect to τ
$(\ )'$	=	indicates differentiation with respect to time

Introduction

ONE of the means by which engineers have coped with the problems of high thermal inputs to aerospace vehicles has been the use of an ablative heat shield Problems involving the transient temperature distribution in bodies undergoing phase changes and the study of the ablative process have thus received considerable attention in the literature In this note, a simple approximate heat-balance technique due to Goodman[1] is utilized to study the ablation of a cylindrical cavity in an infinite medium Further considerations, refinements, and applications of the technique have appeaied in the literature [2–4] A comparison of the heat-balance technique with two other approximate methods was recently reported [5] Lardner[6] discussed the discrepancies between two approximate solutions and known exact solutions to the Stefan problem and the problem of a semi-infinite body under constant heat input

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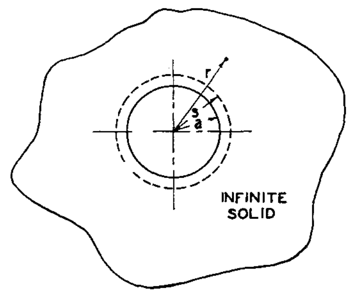

Fig 1 Notation foi coordinate system

Statement of Problem and Assumptions

An infinite medium surrounds a cylindrical cavity The initial temperature T_i throughout the medium is constant, and a heat source in the cylindrical cavity radiates heat to the surface in an axisymmetric manner The initial radius of the cylindrical cavity is a (Fig 1) Time is reckoned from the instant that the heated surface reaches the melt or sublimation temperature T_M For $t > 0$, continued exposure of the heated surface to heat flux causes the material to ablate It is desired to obtain the time history of the ablating heated surface

The following assumptions are made in the analysis: 1) the heated surface remains at the melt or sublimation temperature T_M; 2) the melt or products of sublimation are immediately removed upon formation; 3) the heat flux Q remains constant; and 4) thermal properties of the solid are independent of temperature

The equation governing the axisymmetric flow of heat in the infinite region around the cylindrical cavity is most conveniently written in cylindrical coordinates:

$$k(1/r)(\partial/\partial r)[r(\partial T/\partial r)] = \rho c(\partial T/\partial t) \qquad (1)$$

Equation (1) is valid for $t > 0$, in the region $s(t) < r < \infty$

The initial and boundary conditions to be satisfied are

$$T(s,t) = T_M \qquad (2)$$

$$T(\infty,t) = T_i \qquad (3)$$

$$Q(s) = -k(\partial T/\partial r) + \rho L(\partial s/\partial t) \qquad (4)$$

$$s(0) = a \qquad s'(0) = 0 \qquad (5)$$

It is convenient to introduce the new variables

$$\theta = (T - T_i)/(T_M - T_i) \qquad \tau = \varkappa t/a^2 \qquad (6)$$

$$\zeta = r/a \qquad \xi = s/a$$

The substitution of the variables defined in Eqs (6) into Eqs (1–5) yields the following form of the heat-conduction equation and the associated boundary and initial conditions:

$$(1/\zeta)(\partial/\partial\zeta)[\zeta(\partial\theta/\partial\zeta)] = \partial\theta/\partial\tau \qquad (7)$$

$$\theta(\xi,\tau) = 1 \qquad (8)$$

$$\theta(\infty,\tau) = 0 \qquad (9)$$

$$\alpha - \beta\dot{\xi} = -(\partial\theta/\partial\zeta)_\xi \qquad (10)$$

$$\xi(0) = 1 \qquad \dot{\xi}(0) = 0 \qquad (11)$$

where

$$\alpha = (Qa)/[k(T_M - T_i)] \qquad (12)$$

and

$$\beta = \rho L\varkappa/[k(T_M - T_i)] \qquad (13)$$